Inverse Problems in Electrocardiology
A Computational Approach

Dafang Wang, Mike Kirby, and Chris Johnson
Scientific Computing and Imaging Institute, University of Utah
Overview

Goal:

To reconstruct epicardial surface potentials by measuring body surface potentials

Pipeline:

Graph generated by C. Ramanathan & Y. Rudy
Problem Formulation

The potential field $\Phi$ in human body is modeled by:

$$-\nabla \cdot (\sigma \nabla \Phi) = 0 \quad \text{in } \Omega \quad \text{and} \quad \vec{n} \cdot \sigma \nabla \Phi = 0 \quad \text{on } \Gamma_{\text{torso}}$$

The potential field in 2D domain:

Spatial Distribution on heart

Spatial Distribution on torso
Information Loss

Forward problem:
\[ \Phi_{\text{torso}} = A(\Phi_{\text{heart}}) \]

Inverse problem:
\[ \Phi_{\text{heart}} = A^{-1}(\Phi_{\text{torso}}) \]

Cause of the ill-posedness:
Information loss of the epicardial potential signal

Question:
How to build an optimal computational model which minimizes the information loss, hence improve the accuracy of the inverse problem?
Consider the problem in an annulus domain:

The solution has the form:

\[ \Phi(r, \theta) = a_0 + \sum_{m=1}^{\infty} \left( \frac{r^m}{r^m} + \frac{1}{r^m} \right) \left( a_m \cos(m\theta) + b_m \sin(m\theta) \right), \quad r \in [0.1, 1], \quad \theta \in [0, 2\pi] \]
A Simplified Model Study

Volume conductor refinement
In forward problem, capture high potential gradient, w.r.t. normal direction $r$:

In inverse problem, capture high spatial frequency, w.r.t. azimuthal variable $\theta$:
Discretization Framework

1. Decide the resolution on epicardial surface with \textit{a priori} assumptions

2. Decide the sufficient resolution on body surface
   Fourier analysis of Laplace’s equation

3. Refine the volume conductor
   Any closures that enclose the heart have at least the same resolution as
   that on the epicardial surface
   Fixing the resolution on both boundary surfaces.

Mesh refinement can be:
   h-refinement
   p-refinement
   hybrid of the above
Refinement Technique

Triangle mesh is popular in simulation

Drawback: May yield bad aspect ratio when boundary resolution is fixed

We propose two refinement techniques:

1. Add layers of quad mesh near the boundary
   Quads handle large aspect ratios well

2. Linear truncation scheme from high-order FEM
   Use high-order FEM to approximate volume-conductor,
   but only solve the linear term of epicardial potential
   and discard high-order terms

\[
\begin{bmatrix}
\Phi_T^1 \\
\Phi_T^2 \\
\Phi_T^3
\end{bmatrix} = \begin{bmatrix}
A^{1,1} & A^{1,2} & A^{1,3} \\
A^{2,1} & A^{2,2} & A^{2,3} \\
A^{3,1} & A^{3,2} & A^{3,3}
\end{bmatrix} \ast \begin{bmatrix}
\Phi_H^1 \\
\Phi_H^2 \\
\Phi_H^3
\end{bmatrix}
\]

\[
\Phi_T^1 = A^{1,1} \ast \Phi_H^1
\]
Generalization

Similar results hold for different geometries:
Conclusions & Future Work

Conclusions:
1. A quantitative analysis of the information loss of epicardial potentials
2. A discretization framework to formulate a computationally optimal discrete inverse problem
3. Combined use with other techniques (statistical method, Tikhonov method)

Future Work:
1. Extend the analysis of frequency spectrum to 3D problems.
2. A quantitative assessment of the effect of anisotropic conductivities on the information loss of epicardial potentials
Thanks!
Enjoy the Lunch😊